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2000 J. Phys.: Condens. Matter 12 6851

(http://iopscience.iop.org/0953-8984/12/30/314)

J. Phys.: Condens. Matter **12** (2000) 6851–6858. Printed in the UK PII: S0953-8984(00)10260-7

Electron states in a magnetic quantum ring

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Received 10 December 1999, in final form 12 June 2000

Abstract. Electron states in a magnetic quantum ring are investigated. The electron is assumed to be confined to a plane in the presence of a magnetic field which is zero within the ring $a < r < b$ and has a constant value B_0 outside it. In this way, the perturbation caused by this field distribution is found to be quite different from that corresponding to a magnetic quantum disk, as regards electron states and circulating currents. In the case of electron states, the main effect is the splitting of the corresponding Landau levels into a sort of quasi-miniband around them and modulated by the inner radius. It is also found that probability current can change its direction depending on the missing flux quanta, a mechanism that is also governed by the size of the inner radius. Moreover, when the inner radius approaches zero, the results for a magnetic quantum disk are recovered, as expected.

1. Introduction

Recent advances in fabrication techniques at micrometric or nanometric scales have made possible the additional confinement of a two-dimensional electron gas (2DEG) within structures such as quantum wires and quantum dots, when these structures are subjected to additional potentials like the presence of a magnetic field. Electron transport behaviour in these structures is of special interest since current-carrying edge states formed near the boundary play an important role in resonant tunnelling. An inhomogeneous magnetic field can be obtained in a variety of magnetic structures, which can be synthesized, for instance, by scanning tunnelling microscope lithography [1] or by molecular beam epitaxy [2]. We will use the term 'magnetic quantum ring' for those magnetic structures in which the magnetic field in the plane of the structure has a ring shape.

In usual quantum dots or ring structures, the resonant tunnelling effect gives rise to periodic Aharonov–Bohm oscillations in the magnetoresistance [3, 4], while in a magnetic quantum dot the oscillations in magnetoresistance are found to be aperiodic [5].

In the case of a finite magnetic quantum ring, to our knowledge, little attention has been paid to the characterization of the energy spectrum, its current-carrying states and consequently its transport properties.

In this preliminary report we study the electron states and circulating probability currents due to the inhomogeneous field distribution formed in a magnetic quantum ring. The proposed structure is quite simple—namely, the ring is synthesized by using different materials, which allow it to create an inhomogeneous distribution of magnetic field when a uniform magnetic field is applied. Also, the materials used to synthesize the ring are chosen such that the barrier

0953-8984/00/306851+08$30.00

potentials in the interfaces can be neglected, forming a magnetic field distribution in which the field is zero inside the ring $a < r < b$ and has a constant value B_0 outside it; here a (b) is its inner (outer) radius.

In section 2, the exact solution of the Schrödinger equation for this system is found; the corresponding energy spectrum is calculated as a function of field strength and as a function of angular momentum. In section 3, an analysis of the circulating probability currents is made. Finally, in section 4, some conclusions are drawn.

2. The model

Let us assume a magnetic quantum ring of inner (outer) radius a (b) formed by a magnetic field distribution which is spatially inhomogeneous and is given by

$$\boldsymbol{B} = \begin{cases} B_0\hat{e}_z & 0 \leqslant r \leqslant a \\ 0 & a \leqslant r \leqslant b \\ B_0\hat{e}_z & b \leqslant r < \infty \end{cases} \tag{1}$$

where $\hat{e}_z$ is the unit vector in the z-direction.

The magnetic vector potential associated with this field distribution, which is continuous everywhere, can be written as

$$\boldsymbol{A} = \begin{cases} \dfrac{1}{2}B_0 r\hat{e}_\varphi & 0 \leqslant r \leqslant a \\ \dfrac{1}{2r}B_0 a^2\hat{e}_\varphi & a \leqslant r \leqslant b \\ \dfrac{1}{2r}B_0(r^2 + a^2 - b^2)\hat{e}_\varphi & b \leqslant r < \infty. \end{cases} \tag{2}$$

Notice that $\boldsymbol{\nabla} \times \boldsymbol{A} = 0$ in $a \leqslant r \leqslant b$ and $\boldsymbol{\nabla} \times \boldsymbol{A} = B_0$ for regions $0 \leqslant r \leqslant a$ and $b \leqslant r < \infty$, as is required by equation (1), and also that the symmetric choice $\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$ holds.

In the effective-mass approximation, neglecting the effect of the spin, the Hamiltonian for a single charged particle can be written as

$$H = \frac{1}{2m^*}\left(\boldsymbol{p} + \frac{q}{c}\boldsymbol{A}\right)^2 \tag{3}$$

where m^* is the particle effective mass, q is the absolute value of its charge, $\boldsymbol{p} = -\mathrm{i}\hbar\boldsymbol{\nabla}$ its momentum and $\boldsymbol{A}$ is given by equation (2). Since $\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$, it can be easily seen that for any state vector Φ in Hilbert's space, $(\boldsymbol{p} \cdot \boldsymbol{A})\boldsymbol{\Phi} = (\boldsymbol{A} \cdot \boldsymbol{p})\boldsymbol{\Phi}$—that is, $\boldsymbol{p}$ and $\boldsymbol{A}$ commute; hence

$$H = \frac{\boldsymbol{p}^2}{2m^*} + \frac{qA_\varphi}{m^* cr}L_z + \frac{q^2}{2m^* c^2}A_\varphi^2 \tag{4}$$

where A_φ is the φ-component of $\boldsymbol{A}$ and $L_z = -\mathrm{i}\hbar\,\partial/\partial\varphi$ is the z-component of the angular momentum. The symmetry of H allows one to see that the total wave function $\Psi(r,\varphi) \sim \exp(\mathrm{i}l\varphi)R(r)$; here $l = 0, \pm1, \pm2, \ldots$ is the magnetic quantum number, while $R(r)$ is the radial wave function whose explicit form depends on the region under consideration. After separating the corresponding Schrödinger equation, the differential equations for $R(r)$ in the different regions can be written as

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{l^2}{r^2} - \frac{m^*\omega_c}{\hbar}l - \frac{1}{4}\frac{m^{*2}\omega_c^2 r^2}{\hbar^2} + \frac{2m^* E}{\hbar^2}\right\}R^i(r) = 0 \tag{5}$$

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{s^2}{r^2} + \frac{2m^*E}{\hbar^2}\right\}R^r(r) = 0 \tag{6}$$

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{t^2}{r^2} - \frac{m^*\omega_c}{\hbar}t - \frac{1}{4}\frac{m^{*2}\omega_c^2 r^2}{\hbar^2} + \frac{2m^*E}{\hbar^2}\right\}R^o(r) = 0 \tag{7}$$

where $\omega_c = qB_0/m^*c$ is the cyclotron frequency,

$$s = l + m^*\omega_c a^2/2\hbar$$
$$t = l - m^*\omega_c(b^2 - a^2)/2\hbar$$

and the superscripts (i, r, o) stand for inner, ring and outer regions, respectively. As can be readily seen, the structures of equations (5) and (7) are similar but there is a shortfall in the quantum number l for the outer region which depends on the size of the ring, while within it there is an excess in l which depends only on the inner radius. The latter reflects, qualitatively, that the missing flux quanta in the outer region are mediated by the creation of flux quanta in the ring region; therefore, one can expect important consequences for current transport between outer and inner regions or vice versa; briefly, such a ring structure would constitute an interesting device for modulating electron transport. We shall return to this point later.

When the changes

$$\begin{aligned} u &= \frac{r^2}{2\ell_m^2} \qquad & R^i(u) &= B\exp(-u/2)u^{l/2}X(u) \\ v &= k\frac{r}{\ell_m} \qquad & R^r(v) &= G(v) \\ w &= \frac{r^2}{2\ell_m^2} \qquad & R^o(w) &= D\exp(-w/2)w^{t/2}H(w) \end{aligned} \tag{8}$$

are introduced into equations (5)–(7) (here $\ell_m = \sqrt{\hbar c/qB_0}$ is the magnetic length, $k = \sqrt{2E/\hbar\omega_c} \equiv \sqrt{2\varepsilon}$ and B, D are normalization constants), the resulting equations allow us to recognize that

$$\begin{aligned} X(u) &= {}_1F_1(-\alpha, |l| + 1, u) \\ G(v) &= C_1 J_{|s|}(kv) + C_2 Y_{|s|}(kv) \\ H(w) &= U(-\beta, |t| + 1, w) \end{aligned} \tag{9}$$

where ${}_1F_1$ (U) is the standard (logarithmic) hypergeometric function, $J_{|s|}$ $(Y_{|s|})$ is the Bessel function of the first (second) kind and order $|s|$, C_1 and C_2 are constants,

$$\alpha = \varepsilon - (|l| + l + 1)/2 \qquad \beta = \varepsilon - (|t| + t + 1)/2.$$

Notice that with these changes, distances are now measured in magnetic lengths, while energies are measured in units of $\hbar\omega_c$, both of which depend on field strength.

Now, the energy spectrum of the system can be obtained by requiring the continuity of R^i, R^r, R^o and their derivatives (or equivalently, of X, G, H and their derivatives), across the boundaries of the magnetic quantum ring, i.e.,

$$\left.\frac{1}{R^i}\frac{\mathrm{d}R^i}{\mathrm{d}r}\right|_{r=a} = \left.\frac{1}{R^r}\frac{\mathrm{d}R^r}{\mathrm{d}r}\right|_{r=a} \tag{10}$$

$$\left.\frac{1}{R^r}\frac{\mathrm{d}R^r}{\mathrm{d}r}\right|_{r=b} = \left.\frac{1}{R^o}\frac{\mathrm{d}R^o}{\mathrm{d}r}\right|_{r=b}. \tag{11}$$

These boundary conditions provide an equation whose roots will provide the energy eigenvalues. This equation is of the form

$$\begin{aligned}[][f(u_a)J_{|s|}(v_a) - J'_{|s|}(v_a)][g(w_b)Y_{|s|}(v_b) - Y'_{|s|}(v_b)] \\ + [g(w_b)J_{|s|}(v_b) - J'_{|s|}(v_b)][f(u_a)Y_{|s|}(v_a) - Y'_{|s|}(v_a)] = 0\end{aligned} \tag{12}$$

where

$$\begin{aligned} f(u_a) &= \frac{a}{\sqrt{2\varepsilon}}\left[-\frac{1}{2} + \frac{|l|}{2u_a} + \frac{{}_1F'_1(-\alpha, |l|+1, u_a)}{{}_1F_1(-\alpha, |l|+1, u_a)}\right] \\ g(w_b) &= \frac{b}{\sqrt{2\varepsilon}}\left[-\frac{1}{2} + \frac{|t|}{2w_b} + \frac{U'(-\beta, |t|+1, w_b)}{U(-\beta, |t|+1, w_b)}\right] \end{aligned} \tag{13}$$

and $u_a = a^2/2\ell_m^2$, $v_a = a/\sqrt{2\varepsilon}\ell_m$, $v_b = b/\sqrt{2\varepsilon}\ell_m$, $w_b = b^2/2\ell_m^2$. The primes denote derivatives of the corresponding functions.

The results of finding the roots of equation (12) are displayed in figure 1 as functions of field strength, for a ring with $s_a = a^2/2\ell_{m_1}^2 = 3$, $t_b = b^2/2\ell_{m_1}^2 = 5$ and $l = 0, \pm1, \pm2$ and ±3. Here $\ell_{m_1} \approx 256.5$ Å is the magnetic length at $B_0 = 1$ T. For the sake of clarity, only

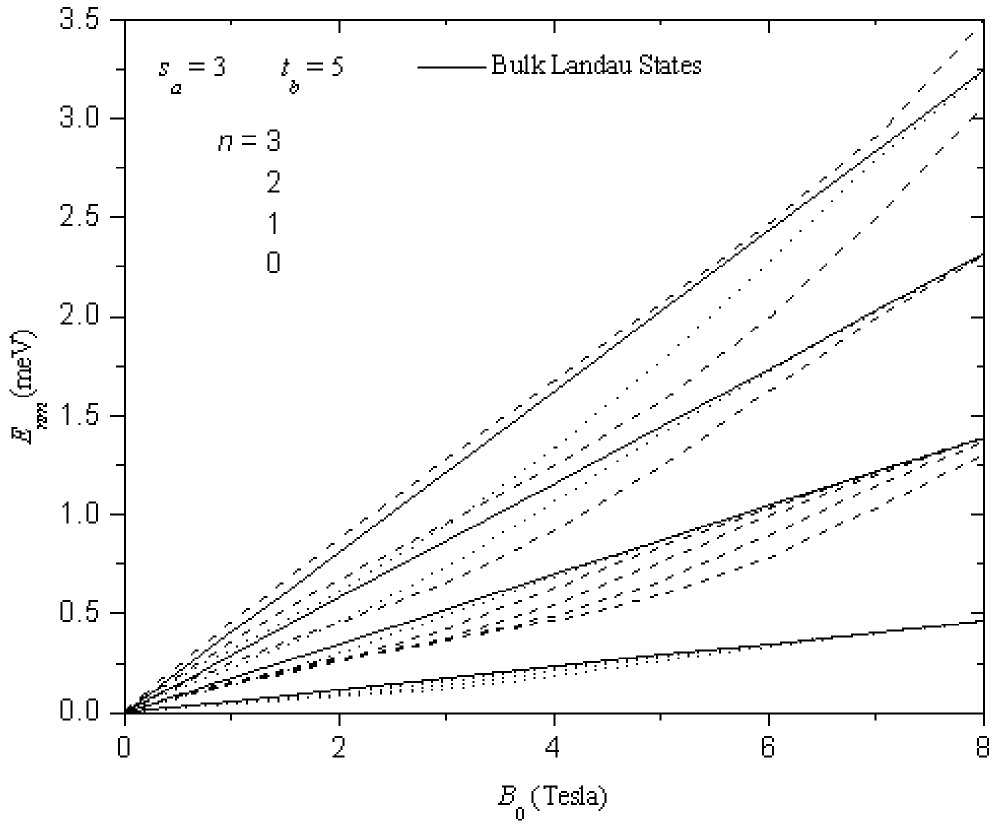

Figure 1. The energy spectrum of a magnetic quantum ring (inner radius a, outer radius b), as a function of field strength. Short-dashed curves correspond to the first root of the eigenvalue equation while solid lines correspond to Landau levels. Long-dashed curves correspond to the second root of the eigenvalue equation. In both cases, the uppermost curves are for $l > 0$ while the lowermost ones are for $l \leqslant 0$.

the lowest two roots are shown. As a first result, notice the modulation in the splitting of the spectrum around the Landau states due to the fact that the number of missing flux quanta (excess flux quanta), for fixed dimensions of the ring, increases with field strength. The latter effect is stronger for states with $l > 0$.

Figure 2 shows the lowest energy level (the first root of equation (12)) as a function of l, for a ring with $t_b = 5$, different values of s_a and a field strength of 2 T. Note the change in sign of the derivatives of the curves around the value of s_a for states with $l < 0$. The latter effect becomes less pronounced as $|l|$ increases. When s_a approaches zero, the results become those for a magnetic disk previously reported, as expected. Recalling that the probability currents $\sim\partial E_{nl}/\partial l$, as a second result we can say that the inner radius defines the direction of current circulation for $l < 0$, while circulation is always anticlockwise for $l > 0$.

In brief, the above results show a quite different physical behaviour for this structure as compared with that of a magnetic quantum disk [5].

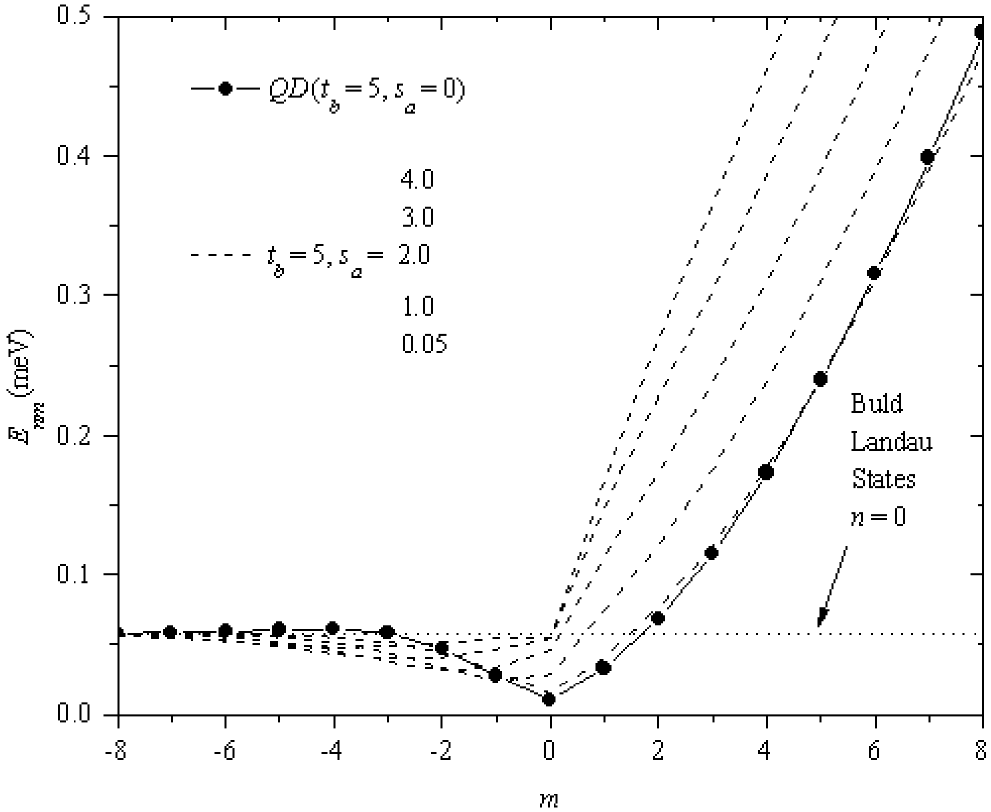

Figure 2. The energy dependence on the angular momentum for a magnetic quantum ring for different sizes of the inner radius and $B_0 = 2$ T.

3. The electron current flow

In this section we shall analyse the electron current flow within the ring. To do this we recall that the total probability current density can be written as

$$j = j_0 + j_A \tag{14}$$

where

$$j_0 = \frac{i\hbar}{2m^*}(\Psi\,\nabla\Psi^* - \Psi^*\,\nabla\Psi)$$
$$j_A = \frac{e}{m^*c}A|\Psi|^2 \tag{15}$$

and Ψ is the wave function given by the multiplying $\exp(il\varphi)$ by the solution of equations (5)–(7).

After a simple calculation, the total probability-current-density vector can be found to be

$$j^i = (l+u)\frac{\hbar|R^i(u)|^2}{\sqrt{2}l_m m^* u}\hat{e}_\varphi$$
$$j^r = (l+u_a)\frac{\sqrt{2\varepsilon}\hbar|R^r(v)|^2}{l_m m^* v}\hat{e}_\varphi \tag{16}$$
$$j^o = (l+u_a-w_b+w)\frac{\hbar|R^o(w)|^2}{\sqrt{2}l_m m^* w}\hat{e}_\varphi$$

where $u_a = a^2/2\ell_m^2$ and $w_b = b^2/2\ell_m^2$.

In analysing the behaviour of the probability currents, we restrict ourselves to evaluating the values in parentheses in equation (16), since the multiplying factors are always greater than or equal to zero.

Thus, as can be immediately seen, in the states with $l \geqslant 0$ the current flow is always anticlockwise (in the 'right direction'), i.e., in the same direction as it would be classically. However, when $l < 0$, j can be in either direction, depending on the dimensions of the ring; i.e., depending on the values of u_a and w_b. Therefore, when $l < 0$, the inner radius is that which in turn determines the direction of current flow. Indeed, three regimes arise from this, namely: $u_a < |l|$, $u_a = 0$ and $u_a > |l|$.

In the first case ($u_a < |l|$), the currents in the inner region and within the ring region are always clockwise since $(l+u) \leqslant (l+u_a) < 0$, while outside the ring there is a transition region in which the current changes its direction from clockwise to anticlockwise, at the point $w_0 = w_b - l - u_a = w_b + |l| - u_a > w_b = b^2/2\ell_m^2$ (see figure 3(a)).

In the second case ($u_a = |l|$) the current is always anticlockwise in the outer region since $w - w_b \geqslant 0$ ($w \geqslant b^2/2\ell_m^2 = w_b$), is zero within the ring ($u_a + l = 0$) and is clockwise in the inner region since $u + l = u - u_a < 0$ ($u \leqslant a^2/2\ell_m^2 = u_a$) (see figure 3(b)).

Finally, in the third case ($u_a > |l|$), the current outside and within the ring is always anticlockwise since $l + u_a - w_b + w > 0$ ($u_a > |l|$, $w \geqslant w_b$) and $u_a - l > 0$; however, in the inner region $l + u = -|l| + u$ can be zero at the point $u_0 = |l| < u_a = a^2/2\ell_m^2$ and thus the current is clockwise when $0 \leqslant u < u_0$, while it will be anticlockwise when $u_0 < u \leqslant u_a = a^2/2\ell_m^2$ (see figure 3(c)).

4. Conclusions

Electron states in a magnetic quantum ring were investigated in this preliminary report. The results show a modulation in the level splitting around the corresponding Landau states, a behaviour which differs substantially from that of a magnetic quantum disk. The probability currents also show a peculiar behaviour and transition regions where the current can change its direction were also found. Both effects on energy states and circulating currents are controlled by the size of the ring's inner radius. The proposed structure would be an interesting candidate for experimental characterization similar to that for a Q1D ring [3], in view of the consequences of its behaviour as regards electron transport. Also, detailed theoretical study of the transport

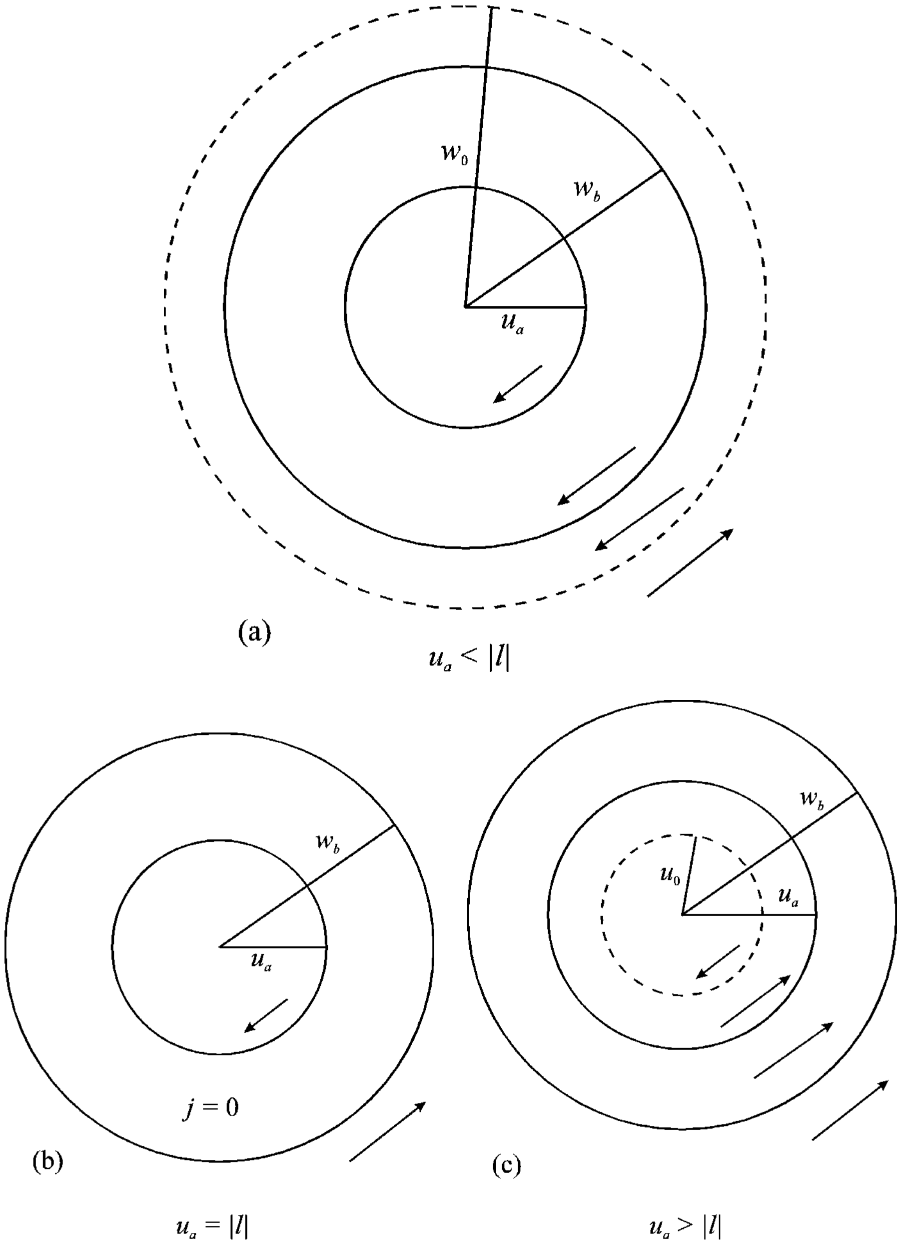

Figure 3. Probability currents in a magnetic quantum ring: (a) $u_a < |l|$, (b) $u_a = |l|$ and (c) $u_a < |l|$.

properties of such a structure would constitute an attractive topic since the trends indicate that it would behave quite differently to its close partners, the magnetic quantum disk [5] and the one-dimensional quantum ring [3], in the light of the results found for probability currents.

The structure proposed herein could give rise to new optical and transport properties of nanostructures, e.g. in SEMM [6] (single-electron MOS memory) or magnetic quantum dots (MQD) [5, 7, 8].

Finally, we hope that this work will motivate detailed experimental research on this kind of system to elucidate whether the predicted behaviour can be observed experimentally or not, since the proposed structure can be fabricated by a technique similar to that used to construct a magnetic quantum dot [1].

Acknowledgment

This work was supported partially by CONACYT (México) under grants 4055-E9403 and 96021.

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